5.5 Notes and Examples

Name: Block: Seat:

Bases other than e and Applications

1. Precalc "Warm up": Solve for x

   (a) $3^x = \dfrac{1}{81}$

   (b) $\log_2 x = -4$

   (c) Half-life is the time it takes for half of the material to decay. When modeling the half-life of a radioactive substance, it is convenient to use $\frac{1}{2}$ as the base for the exponential model. For example, the half-life of Carbon-14 is about 5730 years (actually 5730 ± 40), so if you start with an initial amount A_0 of the substance, then the amount A remaining after t years can be modeled by

   $$A = A_0 \left(\frac{1}{2}\right)^{t/5730}$$

   Often we use $A_0 = 1$ to represent 100%, and then A would be the percentage of what is left.

   1. What percentage of Carbon-14 is left after 2,000 years?

   2. When organic material is "Carbon dated" the percentage of Carbon-14 can be determined by comparing it to the amount of Carbon-12 or Carbon-13 (which are both stable). If we detect that 93% of the Carbon-14 is remaining, how old is the organic material?

2. We now know a lot about e^x and its inverse ______________. To handle any other base a we will use the fourth property of log (the change of base property): $\log_a x =$ ______________

3. Definitions: If $a, x \in \mathbb{R}$ and $a > 0$:

   (a) $a^x =$ ______________

   (b) $\log_a x =$ ______________

   (c) If $a = 10$, instead of writing $\log_{10} x$ we write ______________.

   (d) If $a = 1$, then $y = 1^x$ is the constant function ______________

Derivative Theorems

1. $\dfrac{d}{dx}[a^x] = a^x \ln a$

   Proof: If $f(x) = a^x = e^{\ln a^x} = e^{x \ln a}$ then

   $f'(x) =$ ___________________ $=$ ___________________

2. In general, $\dfrac{d}{dx}[a^u] = (\ln a)a^u \dfrac{du}{dx} = (\ln a)a^u u'$

3. $\dfrac{d}{dx}[\log_a x] = \dfrac{1}{x \ln a}$

   Proof: If $f(x) = \log_a x =$ ___________________ then $f'(x) =$ ___________________ $= \dfrac{1}{x \ln a}$

4. In general $\dfrac{d}{dx}[\log_a u] = \dfrac{u'}{u \ln a}$

4. Derivative Examples

   (a) If $y = 2^x$, then $y' =$

   (b) If $y = 2^{3x}$, then $y' =$

   (c) If $y = \log \cos x$, then $y' =$

(d) If $f(x) = 5^{x^2-2x}$, find $f'(x)$.

(e) If $f(x) = x(4^{-x})$, find $f'(x)$.

(f) If $f(x) = \log_5 \sqrt[3]{2x^2+7}$, find $f'(x)$.*Hint: use the properties of exponents first*

(g) If $f(x) = \log \dfrac{5x^3}{(x^2-3x)^3}$, find $f'(x)$. *Hint: separate the log first*

5. Watch your step: Keep in mind what is a variable, and what is a constant.
   (a) If $y = e^e$, then $y' =$

   (b) If $y = e^x$, then $y' =$

   (c) If $y = x^e$, then $y' =$

   (d) If $y = x^x$, then $y' =$

Integration Theorems

1. $\displaystyle\int a^x \; dx = \frac{a^x}{\ln a} + C$

2. In general, $\displaystyle\int a^u \; dx = \left(\frac{1}{\ln a}\right) a^u + C$

   Proof: Since $a^x = e^{\ln a^x} = e^{x \ln a}$, we can write $\displaystyle\int a^x \; dx =$______________

   Next Let $u = x \ln a$, so that $du =$ ______________

6. (a) $\displaystyle\int 2^x \; dx$

   (b) $\displaystyle\int \frac{3^{-2/x}}{x^2} \; dx$

   (c) $\displaystyle\int_{-1}^{3} 3^x \; dx$

   (d) $\displaystyle\int_{0}^{1} 3^x - 2^x \; dx$